

On the Probabilistic Structure of Quasi-free States of a Clifford Algebra

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Abstract. We prove that the correlation functions of a non-relativistic Fermi field given by a quasi-free state are directly related to the values of the characteristic function of a probability measure over the phase space of a classical spin system.

Introduction

In the past few years probabilistic methods have been proved to be very useful in constructive field theory and especially for the study of Schwinger functions. Indeed the Schwinger functions of a Bose field are symmetric functions of their arguments, hence they can be the moments of a probability measure; moreover for the free Bose field the Wick theorem leads to a gaussian measure [1].

However for fermions and except for some results [2, 3], the situation is less clear, even for the free Fermi field, since the Schwinger functions are completely antisymmetric with respect to their arguments. Consequently it is a priori impossible to repeat for fermions what has been done for bosons.

Our aim with this note is to present an attempt to bypass this difficulty. Namely what we prove is that, given a quasi-free state over a Clifford algebra, the correlation functions are, up to a trivial factor, the values of the characteristic function of a probability measure over the phase space of a classical spin system. [Theorem (2.26).]

Moreover we show that this theorem is the analogue of what can be done for boson systems.

2. Quasi-free States as Given by the Characteristic Function of a Probability Measure

To fix the notations we repeat some definitions and results which can be found in an extended form in [4].