Commun. math. Phys. 48, 97—118 (1976)

Communications in Mathematical Physics

Complex-dimensional Invariant Delta Functions and Lightcone Singularities

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Abstract. Invariant delta functions (including imaginary-mass case) defined in a complex n-dimensional space-time are explicitly calculated in position space. It is proposed to define products of invariant delta functions in the ordinary Minkowski space by analytically continuing the corresponding n-dimensional ones to $n=4$. The (not only leading but also non-leading) lightcone singularities of $[\Delta(x;m^2)]^2$, $\Delta(x;m^2)\Delta^{(1)}(x;m^2)$, and $[\Delta^{(1)}(x;m^2)]^2$ are shown to be unambiguously determined in this way.

1. Introduction

Recently, much attention has been paid to the dimensional regularization method [1]. Though the Minkowski space is of course four-dimensional, Feynman integrals in momentum space can formally be extended to those in a complex n-dimensional space. Supposing that $\mathrm{Re}\, n$ is sufficiently small, one can calculate the latter without encountering ultraviolet divergences. Then one analytically continues the results to $n=4$ and thus obtains regularized Feynman integrals apart from possible poles located at $n=4$.

The purpose of the present paper is to apply this method to singular products in position space. Since invariant delta functions are singular on the lightcone, their naive products are meaningless as distributions. In order to give them reasonable definitions, we propose to use the dimensional regularization method.

In Section 2, we introduce the notion of complex n-dimensional Fourier transform and investigate its basic properties. In Section 3, we explicitly calculate the expressions for invariant delta functions in a complex n-dimensional space-time. In Section 4, the dimensional regularization method is applied to $[\Delta(x;m^2)]^2$, $\Delta(x;m^2)\Delta^{(1)}(x;m^2)$, and $[\Delta^{(1)}(x;m^2)]^2$, and the expressions for their lightcone singularities are explicitly found without encountering poles at $n=4$. Some discussions on our results are made in the final section.

In Appendix A, we present various formulae of Bessel functions which are used in this paper. Appendices B and C are devoted to the calculation needed in the text.