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Relativistic Quantum Theory without Quantized Fields

I. Particles in the Minkowski Space

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Abstract. Peculiarities of symmetrical quantum systems are considered with the aid of the Mackey's induced representations theory. The four-dimensional coordinate representation of the relativistic quantum mechanics suggested by Stueckelberg in 1941 is rederived, reinterpreted and generalized for an arbitrary spin. Then it is applied to introduce the causal propagator as a particle-antiparticle transition amplitude without consideration of a field equation. Finally the theory of relativistic quantum particles interaction is reformulated without an appeal to the concept of quantized fields.

1. Introduction

The present paper is the first one in the series devoted to reformulation of the relativistic quantum theory of particle interactions in terms of elementary particle states with no appeal to the concept of quantized field. The new formulation is based essentially upon the four-dimensional coordinate representation of relativistic quantum mechanics suggested by Stueckelberg at 1941. The symmetry properties of Minkowski space-time and group-theoretical methods are used in the present paper. The next one will deal with the quantum particle theory in the de Sitter space-time. The same methods appear applicable in this case because the de Sitter space possesses a sufficiently large symmetry group.

Relativistic wave functions have been considered by Stueckelberg [1] with the four-dimensional normalization integral

$$\|\psi\|^2 = \int d^4x |\psi(x)|^2 . \tag{1}$$

The theory based on such functions has met difficulties in interpretation and was forgotten. Yet some authors were discussing the unusual relativistic position operator in the last years [2–5], which proved [2] to correspond to the representation, considered by Stueckelberg in [1]. Let us call this operator the Stueckelberg position operator and the corresponding representation—the Stueckelberg one. The equivalent concept of localization was used in an other connection in [6].