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# Linear Response Theory and the KMS Condition

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**Abstract.** The response, relaxation and correlation functions are defined for any vector state $\omega$ of a von Neumann algebra $\mathfrak{M}$, acting on a Hilbert space $\mathscr{H}$, satisfying the KMS-condition. An operator representation of these functions is given on a particular Hilbert space $\tilde{\mathscr{H}}$.

With this technique we prove the existence of the static admittance and the relaxation function. Finally we generalize the fluctuation-dissipation theorem and other relations between the above mentionned functions to infinite systems.

## I. Introduction

In conventional statistical mechanics an equilibrium state of a finite system is given by a Gibbs state. It is well known that the states of infinite continuous systems are no longer of this type. It has been suggested and now widely accepted that an equilibrium state of an infinite system should be described by a state satisfying the KMS-condition [1]. This is also the point of view of this paper.

The problem of non-equilibrium statistical mechanics is to explain the occurrence of an equilibrium state. This problem can be tackled in different ways. There is a direction where people study the problem by placing the system in a larger one. This leads to the study of open systems, where topics like the master equation are widely studied [2, 3], some aspects of the theory have recently been made rigorous [4–6]. Also a lot of rigorous work has been done on models, such as harmonic oscillators and lasers (see e.g. [7]). Another way of studying the problem is to consider small perturbations of the system and to wait for the behaviour after a long time (see e.g. [8, 9]). Linear response theory must be situated in this direction and the principal purpose of the present paper is to prove and generalize to infinite systems rigorously some aspects of linear response theory, as introduced by Kubo [10] and Mori [11].

In Section II we introduce a new scalar product on the set of observables and define a new Hilbert space $\tilde{\mathscr{H}}$, and construct explicity a unitary operator from $\tilde{\mathscr{H}}$ to the KMS-Hilbert space. We prove that it is equivalent with the scalar product of the Kubo-Mori theory, and we give some other characterizations of this scalar product.

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