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Analyticity Properties and Many-Particle Structure in General Quantum Field Theory

II. One-Particle Irreducible n-Point Functions

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Abstract. The extraction of one-particle singularities from the n-point functions is performed in the framework of L.S.Z. field theory in the case of a single massive scalar field. It is proved that the "one-particle irreducible" functions thus obtained enjoy the analytic and algebraic primitive structure of general n-point functions (up to a finite number of generalized C.D.D. singularities). Finally under an additional technical assumption, it is shown that the Glaser-Lehmann-Zimmermann relations stating the completeness of asymptotic states yield similar relations satisfied in any given channel by the corresponding one-particle irreducible functions.

I. Introduction

In the first paper of this series [1] and in a previous work [2], we described a general method for investigating the analyticity properties of the n-point Green's functions implied by the non-linear structure of general quantum field theory.

This non-linear program makes use conjointly of the rigorous results of the linear program expressed in complex momentum space, and of the many-particle structure analysis (M.P.S.A.) due to Symanzik [3] where an essential role is played by the notion of p-particle irreducible (p.i.) part of a Green's function.

The advantage for a synthesis of these two approaches is twofold. On one hand the M.P.S.A. program can be carried out there on a rather rigorous level since it can be developed in the framework of the analyticity properties of Green's functions which is well-established in the L.S.Z. field theory [5–7]. Then it is expected that (apart from certain technical postulates which have to be added to the axioms of the theory) the introduction of irreducible n-point functions can be made rigorous in complex momentum space and that their analytic structure can be clearly exhibited (except for the possible occurence of generalized C.D.D. poles as explained in the following).

On the other hand the use of the non-linear information of field theory, expressed through the *completeness relations* of Glaser, Lehmann and Zimmermann [8], should lead in the M.P.S.A. context to important improvements in the knowledge of the global analytic structure of the n-point functions. Actually the rigorous introduction of the irreducible functions in axiomatic field theory seems to enrich the latter by a powerful algorithm which is borrowed from the perturbative approach [3].