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Some Problems on the Measurement of Quantum Observables and Determination of Joint Entropy in Quantum Statistics

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Abstract. In the paper the equivalency of a successive measurement of observables a and b on the Hilbert space H and a specially organized measurement of the observable $a \otimes b$ on the Hilbert space $\mathscr{H} = H \otimes H$ is determined. The state, wherein the measurement on $\mathscr{H}$ is performed, is shown to be an operator analog of classical joint density of probability distribution. The functionals such as joint and conventional entropy are constructed; the entropy defect of a quantum ensemble and the information quantity contained in quantum measurement are determined.

A number of problems can be formulated where the necessity arises for extension of statistical interpretation of quantum mechanics, construction of the objects of the probability theory such as joint and conventional densities of distribution, determination of functionals generalizing the classical notions of joint and conventional entropy for a quantum case. In particular, these are the problems of the information quantum theory, quantum fluctuation theory, some problems on nonequilibrium statistics. The specific feature of the problems in the list is that the probabilistic distributions (in a classical sense) obtained on the basis of quantum approach are insufficient for their strict formulation. We need to have the operator analogs of joint and conventional distributions, i.e., corresponding generalization of a density matrix.

Realization of the mentioned generalization requires successive inclusion of the measurement operation and measuring instrument in the theory. In the present paper the equivalency of a successive measurement of observables a and b on the Hilbert space H and a specially organized measurement of the observable $a \otimes b$ on the space $\mathscr{H} = H \otimes H$ is determined following the method developed in [1]. The state wherein the measurement is performed on is the above generalization of a density matrix. The latter enables one to determine strictly the notions of joint and conventional entropy in quantum statistics and also the functionals such as the entropy defect of a quantum ensemble and the information quantity contained in quantum measurement.

The initial conceptions for this are the following [1]. The space of self-adjoint trace class operators on Hilbert space H forms a real Banach space V under trace norm. A state space is a triple $(V, K, 1)$ consisting of a real Banach space V, a positive closed cone K which generates V and a linear functional on V such that $1(v) = \langle 1, v \rangle = \|v\|$ for all $v \in V$; V is endowed with a partial ordering by putting