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Absence of Breakdown of Continuous Symmetry in Two-dimensional Models of Statistical Physics

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Abstract. Two-dimensional lattice model is considered. The connected Lie group G acts on a configuration space. The Gibbs potential assumed to be invariant under this action. We prove, that under general assumption on the potential, each Gibbs random field with this potential is also G-invariant.

Introduction

There exist numerous examples of breakdown of discrete symmetry in spin models of statistical physics. Among them we can enumerate the Ising model [1, 2], the anisotropic Heisenberg model [3] etc. In all these examples there is an action of a disconnected group on a configuration space, and the Gibbs potential remains invariant. However, the Gibbs random fields with this potential turn out to be non-invariant under this action.

The goal of the present paper is to show that if the group G is a compact connected Lie group and the field is two-dimensional, the situation is quite opposite. Namely, under some general assumptions, G-invariance of the potential implies G-invariance of each Gibbs random field with this potential.

1. Main Result

Let $\mathbb{Z}^2$ be a two-dimensional lattice with points $t=(k_1, k_2)$, where k_i are integers. Let G be a compact connected Lie group, which acts (on the left) on a space X. In other words, there is a mapping $\pi: G \times X \to X$ with the property $\pi(g_2, \pi(g_1, x)) = \pi(g_2 g_1, x)$, where $g_1, g_2 \in G$, $x \in X$.

Let μ be a G-invariant σ-finite measure on X, defined on a G-invariant σ-algebra $\mathscr{B}_X$ of subsets of X. Let us fix a structure of Riemannian manifold and a two-side invariant Riemannian metric on G. Let $\mathscr{B}_G$ be σ-algebra of Borel subsets of G. The action of G on X is assumed to be measurable, i.e. the mapping $\pi: G \times X \to X$ is measurable. (In the space $G \times X$ the σ-algebra $\mathscr{B}_G \times \mathscr{B}_X$ is considered.)

Let $A \subseteqq \mathbb{Z}^2$ be any subset. Configuration on A is an arbitrary function $x_A: A \to X$; it is determined by its values $x_A = (x_t; t \in A)$, $x_t \in X$. Denote X^A the set of all configurations on A. If $J \subset A$, then $x_J = (x_t; t \in J)$ means restriction $x_A|_J$. On the set of all configurations on A a measure μ_A is naturally introduced as a product of $|A|$ measures μ. (Here and further on $|A|$ denotes the cardinality of A.) μ_A is