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On the Landau Diamagnetism

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Abstract. The grand-canonical partition function of an assembly of free spinless electrons in a magnetic field enclosed in a box (Dirichlet boundary conditions) is shown to be an entire function of the fugacity z and the magnetic field H, as a consequence of the trace-norm convergence of the perturbation series for the statistical semigroup. This allows to derive analyticity properties of the pressure as a function of z and H, and to express the coefficients of its power series expansion around $z=H=0$ by means of the unperturbed semigroup. Hence, the magnetic susceptibility at zero field and fixed density is expressed in terms of Green functions of the heat equation. Its asymptotic expansion for $\Lambda\to\infty$ (Fisher) along parallelepipedic domains is obtained up to $0\left(\frac{S(\Lambda)}{V(\Lambda)}\right)$. The volume term of this expansion is the Landau diamagnetism.

1. Introduction

This paper is concerned with the diamagnetic susceptibility at thermal equilibrium and zero magnetic field of an assembly of free spinless electrons in a box. The problem originates at L. D. Landau [1] in 1930, who gave a treatment in the framework of quantum statistical mechanics, in which, however, the influence of the walls of the container has been considered approximately by a semiclassical argument. His result is different from that obtained in classical statistical mechanics [2], and this gave rise to a debate on the influence of the walls (which is in fact responsible for the null magnetic moment classically obtained). This debate is still alive, because of the many contradictory results obtained. Such a situation is due either to employing approximation procedures hard to control, or to replacing the original problem with a soluble one whose connection with the former is difficult to judge. We shall therefore consider once again the original quantum statistical problem with the proper mathematical rigor.

We shall use grand-canonical quantum statistical mechanics, in which the whole information about the system is contained in the partition function:

$$\Xi_\Lambda(\beta, z, \omega) = \sum_{n=0}^{\infty} z^n \operatorname{tr}\exp[-\beta H_{n,\Lambda}(\omega)] \tag{1.1}$$