Commun. math. Phys. 40, 7—13 (1975)

Ideal, First-kind Measurements in a Proposition-State Structure

G. Cassinelli and E. G. Beltrametti

Istituto di Scienze Fisiche, Università di Genova, I.N.F.N., Sezione di Genova, Genova, Italy

Received May 9, 1974

Abstract. Let $\mathscr{L}$ be an orthomodular lattice and $\mathscr{S}$ a strongly ordering set of probability measures on $\mathscr{L}$ such that supports of measures exist in $\mathscr{L}$. Then we show the existence of a set of mappings of $\mathscr{S}$ into $\mathscr{S}$ that are physically interpretable as ideal, first-kind measurements.

1.

In the conventional formulation [1–3] of the so called logic of quantum mechanics the basic mathematical structure associated to a physical system is the pair $(\mathscr{L}, \mathscr{S})$ where $\mathscr{L}$ is the set of propositions (yes-no experiments), $\mathscr{S}$ the set of states; each $\alpha \in \mathscr{S}$ defines on $\mathscr{L}$ a probability measure $\alpha : \mathscr{L} \to [0, 1]$, and $\alpha(a)$, $a \in \mathscr{L}$, is interpreted as the probability of the yes answer of a, when the initial state of the system is α. $\mathscr{L}$ is given the appropriate structure of orthomodular lattice by means of suitable axioms which elude any requirement on the transformations of the state of the system caused by the measurement of a: the usual postulates of quantum theory of measurement are considered as independent from the structure of $\mathscr{L}$ (for details and further bibliography we refer to [4]).

Pool [5, 6] has suggested an alternative approach which uses as basic mathematical structure the proposition-state-operation triple $(\mathscr{L}, \mathscr{S}, \Omega)$[1], where the operation $\Omega_a \in \Omega$ associated to $a \in \mathscr{L}$ is understood as the transformation of the state of the system induced by an ideal, first-kind measurement (with yes answer) of the proposition a. By use of the postulates of quantum theory of measurement and of the remarkable connections between orthomodular lattices and Baer*-semigroups, he deduces for $\mathscr{L}$ the structure of orthomodular lattice (see also [7]).

In this paper, we shall examine the possibility of reversing the Pool approach: we are going to study whether the assumption of a $(\mathscr{L}, \mathscr{S})$ structure, with $\mathscr{L}$ orthomodular lattice, is sufficient to deduce the

[1] $(\mathscr{L}, \mathscr{S})$ is denoted by Pool as $(\mathscr{E}, \mathscr{S}, P)$ and called an event-state structure; $(\mathscr{L}, \mathscr{S}, \Omega)$ is denoted as $(\mathscr{E}, \mathscr{S}, P, \Omega)$ and called an event-state-operation structure.