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Generalized Quantum Mechanics

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Abstract. A convex scheme of quantum theory is outlined where the states are not necessarily the density matrices in a Hilbert space. The physical interpretation of the scheme is given in terms of generalized "impossibility principles". The geometry of the convex set of all pure and mixed states (called a statistical figure) is conditioned by the dynamics of the system. This provides a method of constructing the statistical figures for non-linear variants of quantum mechanics where the superposition principle is no longer valid. Examples of that construction are given and its possible significance for the interrelation between quantum theory and general relativity is discussed.

1. Introduction

In turn of development of quantum theory efforts were made to present a geometric description of quantum mechanics independent of "wave functions" and "complex amplitudes". The best known such description was originated by Birkhoff and von Neumann [2] and completed by Piron [18]. It explores a partial order relation in a idealized set of "yes-no measurements" called a "quantum logic". The resulting approach though mathematically profound is not physically complete. In the last ten years two other approaches have been developed. One is the algebraic approach reflecting the physics of operations which can be performed on statistical ensembles. This aspect has been introduced to axiomatic quantum field theory by Haag and Kastler [10] and it reappears as the main motif in the present day quantum statistics. The other approach, originated already in the fourties (Segal [1]) might be called "convex". It explores the convex structures of quantum mechanics with a special attention concentrated on the convex set of all states (pure and mixed) of a quantum system. The description of quantum mechanics from that point of view was most systematically explored by Ludwig [14] and further developed in [3–6, 11, 15–17, 19, 21, 22]; it now becomes one of main currents in the foundation of quantum theory. The synthesis of the convex and the algebraic approaches has been gradually achieved [3, 5, 6, 9–11, 16, 19]. It brought the complete geometrization of quantum mechanics including the description of the present day formalism of