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Hilbert Space Theoretic Methods in the Study of Correlation Measures

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Abstract. We study the uniqueness of the association of a classical statistical mechanical state and its sequence of modified correlation measures by studying a cyclic representation of the canonically associated Börchers algebra of the system. We show that semi-analyticity of the cyclic vector for a particular family of symmetric operators is sufficient for uniqueness while analyticity of the cyclic vector permits us to recover the state from the modified correlation measures.

§ 1. Introduction

The problem of the unique specification of a state of a classical statistical mechanical system by an infinite sequence of correlation functions has been studied, in the past, within two widely differing mathematical frameworks. The first is the algebraic approach as typified by the work of Ruelle Ref. [1], and the second is the measure theoretic approach due to Lenard Ref. [2]. In this paper the two approaches are combined.

From the outset we assume the structure, notation, and results of Ref. [2]. Thus the state of a system is represented by a probability measure on the space of locally finite configurations, X, constructed from the one particle space, $(E, \mathscr{R})$, where $\mathscr{R}$ is a ring of subsets of E. We study not correlation functions but correlation measures, the n^{th} correlation measure, ϱ_n, being a measure on $(E^n, \Sigma[\mathscr{R}^n])$ where $\Sigma[\mathscr{R}^n]$ is the σ-algebra generated by $\mathscr{R}^n$.

We construct a "Börchers algebra" from this framework and obtain a representation of it. This is done by introducing "modified correlation measures" which define a "state" on the algebra, i.e. a positive linear functional, and then applying the Gelfand-Naimark-Segel construction. The representation so obtained is cyclic and the operators commute on a dense invariant subspace.

We show that semi-analyticity of the cyclic vector for the operator representatives of the Börchers algebra is sufficient to guarantee uniqueness of the probability measure defining the correlation measures.