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Analyticity Properties Implied by the Many-Particle Structure of the n-Point Function in General Quantum Field Theory

I. Convolution of n-Point Functions Associated with a Graph

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Abstract. This is the first of a series of papers devoted to the derivation of analyticity properties in the non-linear program of general quantum field theory, following the line of the "many particle structure analysis" due to Symanzik. In this preliminary paper, "convolution products" are associated with graphs whose vertices v represent general n_v-point functions. Under convergence assumptions in Euclidean directions, it is proved that any such convolution product H^G associated with a graph G with N external lines is well defined as an analytic function of the corresponding N four-momentum variables. The analyticity domain of H^G is proved to contain the corresponding N-point "primitive domain" implied by causality and spectrum and the various real boundary values of H^G satisfy all the relevant linear relations. For appropriate boundary values, the convolution products generalize the perturbative Feynmann prescription. As a by-product of this study, it is proved that in any perturbative theory using "superpropagators" with Euclidean convergence, Feynmann amplitudes that satisfy all the requirements of the linear program can be defined without the help of a regularization.

1. Introduction

Since the axioms of general quantum field theory were proposed [1, 2], many papers have been devoted to the study of the analyticity properties of the n-point Green's functions of the fields. Interest was particularly taken in the existence of regions of analyticity lying inside the complex mass shell manifold $\mathscr{M}^c$ since in view of the reduction formulae [3, 4] this entails the analytic character of the n-particle scattering amplitudes.

It was by exploiting the so called *linear properties* of the Green's functions that the notion of a "primitive domain" of analyticity of the n-point function in momentum space was derived [5–7]. Then it was soon realized that on one hand this primitive domain did not intersect