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A Laplace Transform on the Lorentz Groups

II. The General Case

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Abstract. We extend the results of a previous paper to arbitrary non-integrable but polynomially bounded functions defined over any connected semi-simple Lie group of real-rank one. Our approach is based on the method of bilateral horospheres and is a direct generalisation of that used earlier. All the features of the more restricted transform are retained in this more general formalism.

Introduction

In a recent paper [1] we introduced a "Laplace transform" on the Lorentz groups $SO(n, 1)$, based on Gel'fand's method of horospheres. That paper was restricted to (nonintegrable) quasiregular or Class I representations, which can be regarded as defined by right translations of functions over the two-sheeeted hyperboloids $SO(n, 1)/SO(n)$; however, a theory of horospheres for the regular representation of an arbitrary connected semi-simple Lie group is now available [2], and in this paper we show how the results of [1] can be extended to arbitrary functions defined over the group itself. Although it is not difficult to treat the quite general case, we shall for simplicity restrict our considerations to the groups of real-rank one, which we shall call the Lorentz groups. These comprise the real Lorentz groups, $SO(n, 1)$; the hermitian, $SU(n, 1)$; the symplectic, $\mathrm{Sp}(n, 1)$; and a real form of F_4 which we shall call the octavian Lorentz group [1].

Our approach is that of [1]: we divide the Fourier transformation into two steps, the integral transform $f(g) \to \hat{f}(h)$, which maps a smooth function over the group into one over the manifold of horospheres, and the Fourier transform over the horospheres themselves. The former we regularize by a process similar to the "analytic continuation in the co-ordinates" used before; the latter we replace by a pair of classical Laplace transforms. We show that the combination of the two, which we call the Laplace transform on G, converges for all polynomially

[1] This remark will not be explained.