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Analyticity and Uniqueness for Spin $\frac{1}{2}$ Classical Ferromagnetic Lattice Systems at Low Temperatures

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Abstract. Analyticity and uniqueness of correlation functions is investigated for a number of systems by application of Ruelle's theorem on zeros of Asano contracted polynomials to the partition function. To answer the question when the partition function of a system is the Asano contraction of those of subsystems the groups appearing in the low and high temperature expansions are employed.

Introduction

In a series of papers that appeared recently ([7, 11, 12], and references given there) a technique generalizing that of Lee-Yang was applied to find regions free of zeros of the partition function, and to deduce analyticity properties of the pressure. The results are obtained mainly for systems with two-body interactions.

The technique consists in decomposing the system in a finite volume into subsystems with uniformly bounded size, when the volume tends to infinity, and in using theorems relating the zeros of the system to those of subsystems. Systematic development of these ideas allows us here to prove analyticity and uniqueness properties of the pressure and the correlation functions of quite general systems.

In applying this method one has to solve two problems. First, one has to show that the partition function of the system can be obtained from those of subsystems by the operation called Asano contraction. Second, suitable information about the zeros of the partition functions of subsystems should be available.

We give a solution of the first problem in terms of groups that appear in the low and high temperature expansions [3, 4, 6, 14]; these groups were studied recently in connection with the duality [6, 14]. In the low temperature region, in which we are interested here, and for ferro-

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