Commun. math. Phys. 33, 171—186 (1973)

The Harmonic Oscillator in a Heat Bath

E. B. Davies

Mathematical Institute, Oxford, U. K.

Received February 16, 1973

Abstract. We study the time evolution of a quantum-mechanical harmonic oscillator in interaction with an infinite heat bath, which is supposed to be initially in the canonical equilibrium at some temperature. We show that the oscillator relaxes from an arbitrary initial state to its canonical state at the same temperature, and that in the weak coupling limit the relaxation is Markovian, that is exponential. In contrast to earlier treatments of the problem [4, 5], the results are obtained without assuming any particular special form for the self-interaction of the heat bath. No use is made of coarse graining, finite memory assumptions or randomly varying Hamiltonians.

1. Introduction

It is well known that for a finite heat bath it is not possible to prove convergence to an equilibrium state in the limit $t \to \infty$ because of the existence of Poincaré recurrences [8, 11, 15]. However, for large systems these recurrences become extremely infrequent and we can eliminate them by passing to the limit of an infinite heat bath. Since the techniques for passing from a finite heat bath to an infinite one are by now well known [2, 5] we immediately consider the Hamiltonian given formally by

$$H_\lambda = H_0 + \lambda H_1 \tag{1.1}$$

where

$$H_0 = \tfrac{1}{2}(p^2 + \omega^2 q^2) + \tfrac{1}{2} : \sum_{n=-\infty}^{\infty} p_n^2 + \sum_{m,n=-\infty}^{\infty} \alpha_{m-n} q_m q_n : \tag{1.2}$$

and

$$H_1 = \sum_{n=-\infty}^{\infty} \gamma_n q_n q \,. \tag{1.3}$$

Here $\{p_m, q_m\}_{m=-\infty}^{\infty}$ are the canonical coordinates of the infinite heat bath and p, q are the canonical coordinates of the oscillator whose time evolution we shall study. We suppose that α is a real symmetric sequence such that for some $\delta > 0$

$$\sum_{n=-\infty}^{\infty} |\alpha_n| \, e^{\delta |n|} < \infty \,. \tag{1.4}$$

As in [5] we must also suppose that α is a positive definite sequence, but we actually suppose slightly more, that

$$\varrho(\theta) \equiv \sum_{n=-\infty}^{\infty} \alpha_n e^{in\theta} > 0 \tag{1.5}$$