Commun. math. Phys. 33, 97—117 (1973)

Absence of Strong Interaction Corrections to the Axial Anomaly in the σ Model

C. Becchi

CERN–Geneva
and
Istituto di Fisica dell'Università, Genova, Sezione INFN di Genova

Received January 29, 1973

Abstract. The absence of strong interaction corrections to the axial anomaly in the σ model is proved in a cut-off independent way using Zimmermann's normal product algorithm.

1. Introduction

In 1969, Adler [1] suggested that there are no higher order corrections to the axial anomaly [1–3]. This suggestion was supported later by Adler and Bardeen [4] with convincing cut-off dependent arguments in the framework of spinor electrodynamics and in a simple version of the Gell-Mann and Lévy σ model [5] coupled to the electromagnetic field.

In the case of the σ model the arguments proposed by Adler and Bardeen are, however, much weaker than in the case of spinor electrodynamics. In fact, Adler and Bardeen do not prove the renormalizability of the model and use Ward identities without being sure that they are not affected by the renormalization procedure. Unfortunately the more relevant case is actually the former because, using the Adler-Bardeen result in the framework of the model, it is possible to compute the low energy value of the $\pi^0 \to 2\gamma$ amplitude.

Recently, Zee [6] and, independently, Lowenstein and Schroer [7] have proved the absence of radiative corrections to the axial anomaly using the Callan-Symanzik equation [8]. In particular the proof given by Lowenstein and Schroer using the Zimmermann's normal product algorithm (NPA) [9] does not involve any cut-off procedure. Using the method of Lowenstein and Schroer we prove in this paper the Adler-Bardeen theorem in the simplified version of the σ model in which the π is an isoscalar meson and only one fermion field (say, the proton field) exists.

The paper is organized as follows. First we state the renormalization rules for the σ model using the NPA (Section 2). Then we derive an equation analogous to the Callan-Symanzik equation for our model