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Local Extensions in Singular Space-Times*

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Abstract. A space-time has a local extension through a point on its b-boundary if and only if an appropriate number of covariant derivatives of the Riemann tensor have limiting values on a curve ending at the boundary-point, measured in a parallely propagated tetrad. The extension has the same differentiability as the space-time if the curve is "reasonable" in a well-defined sense.

1. Introduction

In General Relativity a space-time (M, g) is usually called singular [1] when it is incomplete; the most general sense of this being that it possesses curves κ inextensible at one end with finite length when measured with a parallely propagated tetrad $\{\underset{i}{X}\}$ $\left(\text{i.e. } \int \left[\sum_i \left(g\left(\underset{i}{X}, \dot{\kappa}\right)\right)^2\right]^{1/2} \mathrm{d}s < \infty\right)$. By Schmidt's [2] construction, such a curve defines a point on a boundary that can be attached to M, the b-boundary. Points on the b-boundary can arrise merely from "cutting out" part of a space-time: such points disappear when the removed part is replaced, which leads us to classify boundary points into those which can be so disposed of by an extension, and those for which this is impossible. Only a point of the latter type should be regarded as a true singularity.

Next one seeks to characterise those boundary-points where an extension is possible. In a forthcoming paper [3] Ellis and Schmidt split the problem into two questions. First, is a local extension (to be defined shortly) possible? Secondly, do the local extensions give rise to a true extension? In the present paper we give sufficient conditions for the existence of local extensions of differentiability class C^2 and C^k, where k $(3 \leqq k \leqq \infty)$ is the differentiability of the metric on M. It is shown that, provided the boundary-point is accessible by a curve which is not too pathological (e.g. a causal curve), then a local C^k extension is possible if and only if the components of all relevant covariant derivatives of the

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