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## Axioms for Euclidean Green's Functions

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Abstract. We establish necessary and sufficient conditions for Euclidean Green's functions to define a unique Wightman field theory.

## Contents

1	Introduction
2.	Test Functions and Distributions
3.	The Axioms, Main Theorems
4.	Theorem $E \rightarrow R$
	4.1. Construction of the Wightman Distributions
	4.2. Lorentz Covariance and Spectrum Condition
	4.3. Positivity
	4.4. Cluster Property
	4.5. Locality
5.	Theorem $\mathbf{R} \rightarrow \mathbf{E}$
6.	Arbitrary Spinor Fields
7.	Application
8.	Technicalities

## 1. Introduction

In a relativistic quantum field theory the indefinite metric of Minkowski space causes many problems which could be avoided by replacing the time t by it or the energy E by iE, thereby passing from Minkowski space to Euclidean space. This idea was first used by Dyson [3] in perturbation theory. He continued the Feynman integrands analytically to imaginary energies in order to move the paths of integration away from the mass shell singularities of the causal propagators. Schwinger [21, 22] studied the analytic continuation of time ordered

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