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Local Normality in Quantum Statistical Mechanics

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Abstract. It is shown that K.M.S.-states are locally normal on a great number of C^*-algebras that may be of interest in Quantum Statistical Mechanics. The lattice structure and the Choquet-simplex structure of various sets of states are investigated. In this respect special attention is payed to the interplay of the K.M.S.-automorphism group with other automorphism groups under whose action K.M.S.-states are possibly invariant. A seemingly weaker notion than G-abelianness of the algebra of observables, namely G'-abelianness, is introduced and investigated. Finally a necessary and sufficient condition (on a C^*-algebra with a sequential separable factor funnel) for decomposition of a locally normal state into locally normal states is given.

§ 1. Introduction

The investigation of representations of the commutation relations and anticommutation relations by Dell'Antonio, Doplicher and Ruelle [3] has forced upon us the concept of a locally normal state. Due to this origin, locally normal states have been studied on a C^*-algebra which is a C^*-inductive limit of sub-C^*-algebras, which in essence are irreducible C^*-algebras on suitably chosen Hilbert-spaces [14, 21].

In [12] the concept of a locally normal state has been generalized for a C^*-algebra that is a C^*-inductive limit of a net of von Neumann algebras. In the case where the net consists of factors each having a representation on a separable Hilbert space, the net is called a funnel. We shall use the word funnel for the net of von Neumann (or rather W^*)-algebras that generate a C^*-algebra $\mathfrak{U}$ in the sense of [12], Definition 2, even if the net does not contain only factors and even if the net contains factors which are not of the same type.

It is the aim of this paper to investigate the locally normal character of K.M.S. states on a C^*-algebra $\mathfrak{U}$ with a funnel whose components are σ-finite properly infinite W^*-algebras. As a result we find then that every K.M.S. state on $\mathfrak{U}$ is locally normal. It then follows that ω is normal on every finite factor contained in $\mathfrak{U}$ and on every σ-finite properly