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On the Purification of Factor States

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Abstract. Let $\mathfrak{A}$ be a C^*-algebra and $\mathfrak{A}^\circ$ be an opposite algebra. Notions of exact and j-positive states of $\mathfrak{A}^\circ \otimes \mathfrak{A}$ are introduced. It is shown, that any factor state ω of $\mathfrak{A}$ can be extended to a pure exact j-positive state $\tilde{\omega}$ of $\mathfrak{A}^\circ \otimes \mathfrak{A}$. The correspondence $\omega \to \tilde{\omega}$ generalizes the notion of the purifications map introduced by Powers and Størmer. The factor states ω_1 and ω_2 are quasi-equivalent if and only if their purifications $\tilde{\omega}_1$ and $\tilde{\omega}_2$ are equivalent.

0. Introduction

In the recent paper [4] Powers and Størmer investigated the representations of the CAR algebra $\mathfrak{A}(K)$ induced by generalized free states. Let us recall, that $\mathfrak{A}(K)$ is a C^*-algebra generated by elements $a(f)$ depending linearly on f running over a Hilbert space K and satisfying the canonical anticommutation relations.

Powers and Størmer found that any generalized free state ω of $\mathfrak{A}(K)$ can be extended to a pure state $\tilde{\omega}$ of $\mathfrak{A}(K \oplus K)$. $\tilde{\omega}$ is given by an explicit formula. They proved that generalized free states ω_1 and ω_2 give rise to quasiequivalent factorrepresentation if and only if the representations of $\mathfrak{A}(K \oplus K)$ induced by $\tilde{\omega}_1$ and $\tilde{\omega}_2$ are equivalent. The proof of the last assertion given by Powers and Størmer uses finite dimensional approximations and depends essentially on the particular structure of $\mathfrak{A}(K)$.

In our paper the problem is considered in a more general setting. We prove, that any factor state ω of a C^*-algebra $\mathfrak{A}$ admits extension to a pure state $\tilde{\omega}$ of $\mathfrak{A}^\circ \otimes \mathfrak{A}$ (where $\mathfrak{A}^\circ$ denotes the opposite C^*-algebra) and that correspondence $\omega \mapsto \tilde{\omega}$ obeys all the properties mentioned above.

Now $\tilde{\omega}$ is given by no explicit formula, but is characterized by two requirements: $\tilde{\omega}$ should be j-positive and exact (for details see Section 1).

The exactness of a state $\tilde{\omega}$ is expressed in terms of the representation induced by $\tilde{\omega}$. The condition is not easy to check, but there are some results, like Araki's duality theorem [1], which make it possible in the case of generalized free states.

Let us note, that the existence of pure non exact states is not clear. It is related to the existence of factorisation $F_1, F_2 \subset B(H)$ such that $F_1 \neq F_2'$ (an example of such a factorisation is given in [3]).