Commun. math. Phys. 28, 109—122 (1972)

The Algebra Generated by Physical Filters*

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Received January 31, 1972

Abstract. This paper investigates mathematical properties of a finite-dimensional real algebra of linear operators which are generated by an orthomodular lattice of filters in the sense of Mielnik [4]. Properties of filter decomposability and a representation theorem for the vector space underlying the algebra mentioned are derived.

I. Introduction

The physical background and the motivation of the subsequent mathematical investigations are the papers by Ludwig [3] whose axiom system was, together with the most important mathematical consequences, restated in [2] in a way more adapt to our mathematical considerations. So, referring to [2] for detailed mathematical notes, we will here only sketch basic mathematical concepts in a contemporary language.

A comprehensive and careful analysis of all current attempts of an axiomatic foundation of physical theories has been given by Mielnik [4] who has subordinated the lattice-representing operators $\boldsymbol{T}_E$ of [2] to the physical concept of filters.

II. Preliminaries

We start from a dual pair (B, B') of two real topological vector spaces. As in [2] B (and hence B') are supposed to be finite-dimensional, say $\dim B = \dim B' = N$.

1. B has an order base K which is convex and closed. The elements of K are denoted by V, the elements of B in general by X.

2. In B there exists a proper positive generating cone B_+ generated by K, i.e.

$$B = B_+ - B_+ , \; B_+ = \bigcup_{\lambda \in \boldsymbol{R}_+} \lambda K .$$

3. B' is partially ordered by

$$Y_1 \leqq Y_2 :\Leftrightarrow \langle V, Y_1 \rangle \leqq \langle V, Y_2 \rangle \quad \text{for every} \quad V \in K .$$

* This paper leans on a report presented to and supported by the Deutsche Forschungsgemeinschaft.