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Nelson's Symmetry and the Infinite Volume Behavior of the Vacuum in $P(\phi)_2$*

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Abstract. Let H_l be the Hamiltonian in a $P(\phi)_2$ theory with sharp space cutoff in the interval $(-l/2, l/2)$. Let $E_l = \inf\sigma(H_l)$, $\alpha(l) = -E_l/l$, and let Ω_l be the vacuum for H_l. We discuss properties of $\alpha(l)$ and Ω_l. In particular, as $l \to \infty$, there are finite constants $\beta_\infty < 0$ and α_∞ such that $\alpha(l) \uparrow \alpha_\infty$, $(\alpha(l) - \alpha_\infty)\, l \downarrow \beta_\infty$, and hence $\alpha(l) = \alpha_\infty + \beta_\infty/l + o(l^{-1})$. Moreover $\exp(-c_1 l) \leqq \|\Omega_l\|_1 \leqq \exp(-c_2 l)$ for c_1, c_2 positive constants, where $\|\Omega_l\|_1$ is the $L^1(Q, d\mu_0)$ norm of Ω_l with respect to the Fock vacuum measure. We also present a new proof of recent estimates of Glimm and Jaffe on local perturbations of H_l in the infinite volume limit.

§ 1. Introduction

In this paper, we deal with the by now standard $P(\phi)_2$ field theory [4]. A polynomial $P(X)$ which has real coefficients and which is bounded from below will be called *semi-bounded.* If $P(0) = 0$ and $P \not\equiv 0$, we will say P is *normalized.* Our spatially cutoff Hamiltonian will have a sharp space cutoff. We fix P semibounded and let $H_l = H_0 + V_l$ where

$$V_l = \int_{-l/2}^{l/2} :P(\phi(x)):dx$$

and where H_0 is the free Hamiltonian of mass $m_0 > 0$. By using techniques of "Markov field theory", Nelson recently proved [10]:

$$\langle \Omega_0, e^{-tH_l}\Omega_0 \rangle = \langle \Omega_0, e^{-lH_t}\Omega_0 \rangle \tag{1}$$

where Ω_0 is the Fock ($\equiv H_0$) vacuum. While this space-time symmetry looks innocent, it is extremely deep; in particular, it has the "exponential decoupling of distant regions" built into it via the exponential bound on the semigroup. The usefulness of (1) was noted by Nelson [10] who used it to prove the "linear lower bound" of Glimm-Jaffe [2]: $E_l \equiv \inf\sigma(H_0 + V_l) \geqq -cl$ for some c. In [8] Guerra realized the possibility

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