Commun. math. Phys. 27, 1—9 (1972)

Quadratic Form Techniques and the Balslev-Combes Theorem

Barry Simon*

Departments of Mathematics and Physics, Princeton University, Princeton, N.J., USA

Received January 24, 1972

Abstract. We extend the theorem of Balslev and Combes on the absence of singular continuous spectrum to a class of interactions including $r^{-\alpha}$ $(3/2 \leqq \alpha < 2)$ local potentials. In addition, we note that the theory of sectorial operators allows a simplification of their proof and allows one to push the cuts through angles larger than the $\pi/2$ restriction employed by Balslev-Combes.

§ 1. Introduction

In [1], Balslev and Combes introduced a powerful new technique to the mathematical theory of N-Body Schrödinger operators. This technique has already been used to prove the absence of singular continuous spectrum in the Hamiltonian of certain N-body systems [1] and to study time dependent perturbation theory [11, 12]. Our main goal in this brief note is to show that the Balslev-Combes results can be extended to some other systems; in particular, to systems with central two body potentials, $V_{ij}(r)$, so that V_{ij} has an analytic continuation to $\{r \mid |\arg r| < \alpha\}$ with $W_\theta(r) = V(e^{i\theta} r)$ in the Rollnik class [9], $R + (L^\infty)_\varepsilon$, if $|\mathrm{Im}\,\theta| < \alpha$. The result is a sufficiently straight forward merging of the techniques of [10] with those of [1] that Section 4 where we present the proof will be brief.

Our main goal in [9, 10] was to use quadratic form techniques to extend the theory of two body quantum mechanics to a large class of potentials. But we also found that form techniques could be used in simplifying the proofs and extending the theorems for the Kato classes. This is also true for the Balslev-Combes theory. Not only can we use form techniques to extend their theorem to a larger class (§ 4) but we will show how notions from the theory of sectorial operators allow one to eliminate a difficult technical step from their induction (§ 3). In addition, we will show that the $|\mathrm{Im}\,\theta| < \pi/4$ conditions that they place on their results are artifacts of the way they use Ichinose's lemma and that

* A. Sloan Foundation Fellow.