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Quantum Theory of Particles with Spin Zero and One Half in External Fields*

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Abstract. The unitary (pseudo unitary) time-evolution operator for a particle with spin half (zero) in an external time-dependent electromagnetic (scalar) field is used to generate a Bogoliubov automorphism on the algebra of the free in field. For the case of an electric external field (scalar field) a finite expression for Ω_{out} is given and the S-matrix constructed. The latter is unitary and implements the Bogoliubov automorphism. Theorems by Shale and Stinespring are rederived.

1. Introduction

Numerous papers have been devoted to the subject of quantum theory of particles with spin zero and one half in external fields. The formal aspects were well developed twenty years ago in particular through the work of Feynman [1], Salam and Mathews [2], and Schwinger [3]. A mathematical treatment of the theory is the purpose of this article.

Several authors have prepared ground for such an attempt. Capri [4] explains lucidly the "reduction to a c-number problem" (Section 4 and 5), Boongarts [5] treats at length the case of a spin 1/2 oarticle in a stationary external electromagnetic field. Verifying the assumptions of a theorem by Shale and Stinespring [6] he proves existence of a time evolution operator in Fock space under conditions essentially the same as the ones in Theorem 6 for the case of time dependent external fields. In a previous paper [7] the existence of an out vacuum in the Fock space of the infield is discussed and assumptions necessary for the out-vacuum to exist are verified for the first model to be analyzed below (Section 4).

In the second and the third section the classical theory of a spin zero and spin 1/2 particle in an external field is reviewed to the extent necessary for quantization. Specifically we consider two models, the

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