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On the Directional Dependence of Composite Field Operators*

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Abstract. The Wilson expansion of the field operator product $A_1(x_1)\,A_2(x_2)$ may be used to define composite operators which are local with respect to $\frac{1}{2}(x_1+x_2)$ and depend in addition on a vector η proportional to the distance $x_1 - x_2$. It is proved that the composite operators are polynomials in η, for fixed $\eta^2 \neq 0$, and that their dependence on η^2 only involves powers of η^2 and $\lg\eta^2$.

1. Introduction

The composite operators associated with the formal product $A_1(x)\,A_2(x)$ of two fields may be conveniently defined as the operators C_j appearing in the Wilson expansion

$$A_1(x_1)\,A_2(x_2) = \sum_{j=1}^{k} f_j(\varrho)\,C_j(x,\eta) + P_{k+1}(x,\eta,\varrho)\,, \qquad x_1 = x + \varrho\eta\,, \quad x_2 = x - \varrho\eta\,, \quad \varrho > 0\,, \tag{1.1}$$

where the coefficients f_j satisfy

$$\lim_{\varrho\to 0} \frac{f_{j+1}(\varrho)}{f_j(\varrho)} = 0\,, \qquad \lim_{\varrho\to 0} \frac{P_{k+1}(x,\eta,\varrho)}{f_k(\varrho)} = 0\,. \tag{1.2}$$

In a recent paper [1] the expansion (1.1) was derived from general assumptions, and the operators C_j were shown to be local in x.

The operators C_j depend on the vector x of the center-of-mass point and an additional four vector η proportional to the distance of the arguments x_1 and x_2. The dependence on η is related to the directional dependence of composite field operators. This can be seen by setting

$$\eta = \frac{\xi}{\sqrt{-\xi^2}}\,, \qquad \varrho = \sqrt{-\xi^2}$$

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