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Non-Existence of Goldstone Bosons with Non-Zero Helicity

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Abstract. In a local relativistic quantum field theory a conserved covariant tensor current may lead to a spontaneously broken symmetry if it generates zero mass states from the vacuum (Goldstone theorem). Here it is shown that in addition it is necessary that these massless states have helicity zero if the underlaying state space has a positive metric.

1. Introduction

The well-known Goldstone-theorem in quantum field theory [1] states a connection between the symmetries of a dynamical system and its excitation spectrum. It says that a local conservation law, expressed by the condition $\partial_\mu j^\mu(x) = 0$ for some local current[1], leads inevitably to a global one, expressed by the invariance of the vacuum state under a corresponding group of transformations, if there is a gap above the vacuum in the energy spectrum of the system. Conversely, if $\partial_\mu j^\mu(x) = 0$ but the vacuum is not invariant (in this case we speak of a spontaneously broken symmetry), there must exist excitations of the system with an energy arbitrarily close to that of the vaccum state. In a relativistic theory which we want to deal with in the following, one knows more: There have to be zero-mass states in the state space of the system which can be created from the vacuum by the application of the current operator j_μ. These massless "Goldstone-particles" must have the quantum numbers carried by the current j_μ. We may ask if there is some principal restriction on these or not. Clearly there is none on the internal quantum numbers, since we can easily construct an example of a Goldstone-particle with arbitrary internal quantum numbers with the help of a corresponding massless free scalar field ϕ with $j_\mu(x) = \partial_\mu \phi(x)$ the transformation being the addition of a constant to ϕ. What remains open is the spin (i.e., helicity) of the Goldstone-particle. In order to obtain Goldstone-particles with non-zero spin, the current which creates them from the vacuum has to have additional Lorentz-indices besides its

[1] We want to emphasize that we are only dealing with translation-covariant currents, i.e., $\mathscr{U}(1, a)\, j_\mu(x)\, \mathscr{U}^{-1}(1, a) = j_\mu(x + a)$.