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On Event Horizons in Static Space-Times*

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Abstract. A proof of the (vacuum) Israel theorem on event horizons in static space-times is given employing the Newman-Penrose formalism. The theorem is extended to include the case of a static, massive, complex, scalar field.

1. Introduction

In recent years properties of event horizons have been studied in some detail. Israel [1, 2] has shown that among the asymptotically flat, static, vacuum fields only the Schwarzschild solutions with $m>0$, and among the corresponding electrovac space-times only the Reissner-Nordström solutions with $m^2 \geqq \gamma e^2$ have regular event horizons. Chase [3] has found that in the presence of a static, asymptotically flat, massless scalar field the event horizon has to be singular.

It has been conjectured [1, 2, 4], and Carter [5] has essentially proved for the case of axial symmetry, that among the asymptotically flat stationary vacuum space-times only the Kerr solutions with $m \geqq a$ have nonsingular event horizons. Unfortunately, Israel's proof of the static case, relying heavily on a three-dimensional formalism, does not easily generalize to the stationary case (where the Killing field is not orthogonal to a family of hypersurfaces). In Sections 3 and 4 of the present paper we give a fairly straightforward proof of the vacuum Israel theorem using the well-known Newman-Penrose formalism [6]. Minimal use is made of the hypersurfaces. It is hoped that this method will generalize to the stationary case. Moreover, the differential equations derived in Section 3 should prove useful in solving various other problems involving static fields.

In Section 5 we extend the horizon theorem to include the case of a general (possibly massive and complex) scalar field. Every such field which is gravitationally coupled, static, and asymptotically flat is found to become singular at a simply connected event horizon.

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