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On the Stability of the Taub Universe

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Abstract. An analysis of the stability of the Taub universe for arbitrary, initially small perturbations is carried out. It is found that the perturbations decrease during the expansion and increase during the contraction of the unperturbed space. In the process we obtain the general solution to a system of six coupled, linear, partial differential equations in six unknown functions of four variables.

I. Introduction

The solution to Einstein's field equations found by Taub [1] has aroused considerable interest on several counts. (a) It is a *non-flat* solution of the *empty-space* equations, having closed, homogeneous, space-like hypersurfaces which expand anisotropically. It can thus be interpreted as describing a universe containing nothing but gravitational radiation. (b) It is well behaved for all finite values of the coordinates, but becomes singular (in the sense that certain components of the Riemann tensor become infinite) at $t = \pm\infty$. The invariants $R_{ijkl}R^{ijkl}$ and $R_{ijkl}R^{ikjl}$, however, remain finite. (c) Newman, Unti, and Tamburino [2] have obtained a metric which extends the solution to values of the proper time outside the range covered by Taub's coordinates[1]. (d) The extended space-time is *maximal* [3] (i.e. not part of a still larger space-time), but it is *geodesically incomplete* [3] (i.e. there exist in it geodesics which cannot be extended to infinite values of their affine length). (e) In the part of the manifold outside the two singularities (NUT space) there exist closed time-like curves [4].

Some of these properties are more than mere mathematical curiosities. In particular the singularities and the expansion are features found in almost all cosmological solutions. With its anisotropy, Taub space thus appears to be the simplest generalization of the Friedman models – albeit without matter. In the early stages of the evolution of the universe, however, when the curvature is high, the presence or absence of matter

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[1] The proper time interval covered by Taub's coordinates is finite.