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The Monodromy Rings of One Loop Feynman Integrals

G. PONZANO, T. REGGE, E. R. SPEER*, and M. J. WESTWATER*
Institute for Advanced Study, Princeton, New Jersey

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Abstract. The monodromy rings of Feynman integrals for one loop graphs with an arbitrary number of lines are determined.

§ 1. Introduction

This paper is the second of a series whose general aims were outlined in the introduction to the first paper [1]. In this paper we make a systematic study of the Feynman integral for a general one loop graph in an arbitrary space-time dimension; we classify the possible paths of analytic continuation, label the determinations of the function over a fixed base point, and obtain explicit formulae for the action of analytic continuation on the vector space spanned by these determinations.

A preliminary account of these investigations has been published [2]. The present account is self contained, but for the reader who is familiar with [2], we proceed to give a brief comparison. In the present paper we introduce the auxiliary complex parameters also used in [1] and [3]. This makes a technical difference in the construction of the representation since, for general values of the parameters, the Cutkosky-Steinman relations do not hold; nevertheless, we are able to determine the representation by exploiting more fully the consequences of the Picard-Lefschetz theorem. Single loop integrals fall naturally into four classes (see § 2). In [2] only case 3 (in which no second kind singularities appear, and the invariants constructed from the external momenta are algebraically independent) was treated. Our present use of complex parameters enables us to deal with four cases in a unified manner. Finally, we note that the present paper includes some proofs which were omitted from [2].

We would like to call attention to two points which were raised in the discussion following a talk given by one of us (T. R. [4]). First there is the problem which arises in case 4 – that the scalar invariants of the external momenta are algebraically dependent. We show in this paper

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