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Boson Fields with Bounded Interaction Densities*

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Abstract. We consider interaction densities of the form $V(\phi(x))$, where $\phi(x)$ is a scalar boson field and $V(\alpha)$ is a bounded real continuous function. We define the cut-off interaction by $V_{\varepsilon,r} = \int\limits_{|x|<r} V(\phi_\varepsilon(x))$, where $\phi_\varepsilon(x)$ is the momentum cut-off field. We prove that the scattering operator $S_{\varepsilon,r}(V)$ corresponding to the cut-off interaction exists, and we study the behavior of the scattering operator as well as the Heisenberg picture fields, as the cut-off is removed.

I. Introduction

In two earlier papers [2, 3] we studied self-interacting scalar Boson fields with interaction densities of the form $V(\phi(x))$, where $V(\alpha)$ is a bounded continuous real function. In Ref. [2] we proved that for the corresponding cut-off interaction the asymptotic limits of the fields existed. In Ref. [3] we proved that the Heisenberg picture fields existed as weak limits of the Heisenberg picture fields corresponding to the cut-off interactions. In Section 2 of this paper we prove that the Heisenberg picture fields are trivial in the sense that they are free fields. In Section 3 we prove that the scattering operator $S_{\varepsilon,r}(V)$ corresponding to the cut-off interaction exists, and we prove that the limit as ε tends to zero is 1 if r is small and fixed.

II. The Heisenberg Picture Fields

Let $\mathscr{F}$ by the Fock space of a free scalar boson field $\phi(x)$. The field operators are given in terms of the annihilation-creation operator a^* and a by

$$\phi(x) = 2^{-\frac{1}{2}}(2\pi)^{-\frac{3}{2}} \int\limits_{R^3} e^{ipx}(a(p) + a^*(-p))\,dp\,. \tag{2.1}$$

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