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A Class of Algebraically Special Perfect Fluid Space-Times*

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Abstract. Solutions of the Einstein field equations are considered subject to the assumptions that (1) the source of the gravitational field is a perfect fluid, (2) the Weyl tensor is algebraically special, (3) the corresponding repeated principal null congruence is geodesic and shearfree. If in addition, the repeated principal null congruence is non-expanding, it follows that the twist of this congruence must be non-zero (for a physically reasonable fluid). The general line element subject to this additional restriction is derived. Furthermore, it is shown that all solutions of the Einstein field equations which satisfy (1) and exhibit local rotational symmetry, necessarily satisfy (2) and (3).

§ 1. Introduction

The Einstein field equations with a perfect fluid as source read

$$R_{ab} - \tfrac{1}{2} R g_{ab} + C g_{ab} = -[(A+p)\, u_a u_b - p g_{ab}], \qquad u_a u^a = 1\,, \tag{1.1}$$

where u_a is the velocity of the fluid, $A > 0$ the energy density, p the scalar pressure and C the cosmological constant. The metric tensor g_{ab} $(a, b, c \ldots = 1, 2, 3, 4)$ has signature $(- - - +)$ and the conventions for the Riemann and Ricci tensors are

$$v_{a;bc} - v_{a;cb} = v_d R^d{}_{abc}, \qquad R_{ab} = R^c{}_{abc}\,.$$

Exact solutions of (1.1) are of interest in the following connections:

(1) as cosmological models,

(2) as interior solutions to be matched to exterior vacuum solutions,

(3) as representing the propagation of gravitational radiation in matter.

Algebraically special solutions of (1.1) (i.e., solutions in which the Weyl tensor has a repeated principal null direction, cf. for example Pirani [1]) have recently been studied in each of these three contexts.

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