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The Physical States of Fermi Systems

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Abstract. A classification of all translationally invariant states over the algebra of anti-commutation relations which satisfy criteria of finite mean density, finite mean kinetic energy, and finite mean entropy is given. It is demonstrated that these concepts can be discussed in terms of affine, semi-continuous, functionals which respect the barycentric decompositions of invariant states. Many other pertinent results, both local and global, are derived.

1. Introduction

It is generally accepted that the states of physical systems can be theoretically identified with the mathematical states over suitably chosen C^*-algebras, the algebras of observables. For reasons of economy, however, one usually restricts attention to the subset of states which satisfy general conditions characteristic of the particular physical setting under consideration. In particular in statistical mechanics it is to be expected that a broad enough description is provided by states satisfying requirements of spatial homogeneity together with certain density restrictions. Following the original papers [1–3], the homogeneity requirements have been extensively studied to give a classification of invariant, periodic, and almost-periodic states. The density restrictions, which were particularly emphasized by Ruelle [3], have however attracted less attention. It is the purpose of the present paper to attempt to fill this gap by giving a detailed discussion of these restrictions in the case of Fermi systems. We consider the particle, kinetic energy, and entropy densities.

Each state ϱ over a C^*-algebra $\mathfrak{A}$ determines a representation π_ϱ of $\mathfrak{A}$ on a Hilbert space $\mathscr{H}_\varrho$ and a cyclic vector Ω_ϱ such that

$$\varrho(A) = (\Omega_\varrho, \pi_\varrho(A)\Omega_\varrho)\,.$$

If $\mathfrak{A}$ is the algebra associated with the canonical anti-commutation relations then the concept of particle density is usually discussed in terms