Commun. math. Phys. 14, 195—204 (1969)

A Proof of the Existence of Phase Transitions in the Anisotropic Heisenberg Model

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Received June 13, 1969

Abstract. It is rigorously proved that the anisotropic Heisenberg spin model, in two or more dimensions, exhibits a first order phase transition at low temperatures and for large anisotropy (strong coupling of the third components of the spins).

1. Introduction

It is now universally believed that the usual formalism of equilibrium statistical mechanics is structurally rich enough to both predict phase transitions and give information concerning their nature. Unfortunately it is difficult to justify this belief by rigorous mathematical arguments and theoretical proofs of the existence of phase transitions are indeed scarce. In classical statistical mechanics proofs exist for spin systems, or lattice gases, under a variety of different conditions but the only quantum mechanical system for which a transition has been established is the non-interacting Bose gas in three or more dimensions. The purpose of this note is to provide an existence proof for an interacting quantum system, the anisotropic Heisenberg model in two or more dimensions.

The method we use is a variant of an argument devised by Peierls [1] to prove the existence of a first order phase transition in the two dimensional Ising model. In the framework of classical spin systems this method has been generalized and extended by several authors [2–4]. The first point of this note is to present this argument in a manner applicable to quantum spin systems; this requires a slight reformulation of the method in terms of operators but no essential change. Secondly this reformulation is applied to the anisotropic Heisenberg model. This part of our calculation involves estimates of the norms of certain products of non-commuting operators and to establish these estimates we consider the Heisenberg model as a perturbed form of the Ising model, i.e. the corresponding classical system. It is this perturbation theoretic calculation that restricts our results to a range of anisotropies for which the third components of the spins are strongly coupled.