

# A Local Relativistic Boson Field with the $\lambda|\varphi|$ Interaction

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**Abstract.** We prove that the Heisenberg picture fields for a self interacting Boson field with the  $\lambda|\varphi|$  interaction in four space time dimensions exists as weak limits of Heisenberg picture fields corresponding to the cut-off interactions.

## 1. Introductions

In an earlier paper [4], here after referred to as paper I, we studied self interacting Boson fields with interaction densities of the form  $V(\varphi(x))$  in four space time dimensions, where  $V$  was a bounded continuous function with a bounded uniformly continuous first derivative. We proved in I that the Heisenberg picture fields existed as weak limits of the Heisenberg picture fields corresponding to the cut-off interactions.

The purpose of this paper is to show that similar methods as these used in I, may also be used to prove existence of Heisenberg picture fields for more singular interaction densities. For this reason we shall study the self interacting Boson field  $\varphi(x)$  in four space time dimensions, with the formally local and relativistic invariant interaction

$$\lambda \int_{R^3} |\varphi(x)| dx$$

where  $|\varphi(x)|$  is the absolute value of  $\varphi(x)$ . As in I we introduce the cut-off interaction

$$V_{\varepsilon,r} = \lambda \int_{|x| \leq r} |\varphi_{\varepsilon}(x)| dx,$$

where  $\varphi_{\varepsilon}(x)$  is the momentum cut-off field, and prove that the Heisenberg picture fields corresponding to the cut-off interaction converges weakly as the cut-off is taken away.

## 2. The Cut-Off Interaction

We shall use the Fock space representation. The Fock space  $\mathcal{F}$  is a Hilbert space where the elements are sequences of functions  $f = \{f_0, f_1, \dots\}$  where  $f_n(p_1, \dots, p_n)$  is a symmetric function of  $n$ -variables  $p_1, \dots, p_n$  with