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Boson Fields Under a General Class of Local Relativistic Invariant Interactions

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Abstract. We consider a boson field $\varphi(x)$ under an interaction of the form $\int_{R^3} V(\varphi(x))\,dx$, where $V(\alpha)$ is a bounded continuous real function of a real variable α. If $V(\alpha)$ has a uniformly continuous and bounded first derivative, we prove that the Heisenberg picture field exists as weak limits of the Heisenberg picture fields corresponding to the cut-off interaction.

1. Introduction

The object of this paper is to study a general class of quantum fields with a local relativistic invariant interaction in four space time dimensions. The fields will be self interacting boson fields, with energy operator of the form

$$H = H_0 + \int_{R^3} V(\varphi(x))\,dx\,.$$

H_0 is the free energy operator of a free boson field $\varphi(x)$ of strictly positive mass m. $V(\alpha)$ is a real function of a real variable α, such that $V(\alpha)$ is bounded, continuous and with a bounded and uniformly continuous first derivative.

In two space time dimensions Glimm [1] has investigated the case where $V(\alpha)$ is a polynomial containing only terms of even degree and a positive leading coefficient. For this case he proves that, after renormalization of the interaction by introducing the Wick product, the total energy with a space cut off interaction becomes a semi bounded symmetric operator on the Fock space. The case $V(\alpha)=\lambda\alpha^4$ and still in two space time dimensions, can be treated more thoroughly, as shown by Glimm and Jaffé [4]. Glimm was also able to treat the case $V(\alpha)=\lambda\alpha^4$ in three space time dimensions [2]. The author's reason for studying interactions given by bounded continuous functions instead of polynomials, is strictly that of mathematical convenience, and he hopes that may be in this way enough experience can be gained, so that later on one may be able to treat more realistic models.