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On the Šilov Boundary of the Vertex Function* **

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Abstract. The Šilov boundary of the vertex function is computed without resource to analytic completion techniques.

1. Introduction

A possible systematic approach to field theory is to derive analytic expressions (integral representations) for the various Green functions. These should express the content of the linear axioms, namely, Lorentz-invariance, energy-momentum spectrum, and local-commutativity. The integral representations should then be substituted into the non-linear unitarity [1] or positive definiteness [2] relations for further investigation.

The linear axioms imply analyticity of the Green functions in certain permuted domains [3] $-D$. They may generally be continued into $E(D)$ – the envelope of holomorphy [4] of D. One then tries to set up a generalized Cauchy integral representation for the functions analytic in $E(D)$, which is the desired expression. Now, the Šilov boundary of a domain is the smallest subset of the domain on which one can hope to represent a holomorphic function by an integral representation. Hence the Šilov boundary is all that is actually needed. It is known [5] that the Šilov boundary of a domain $-S(D)$ coincides with that of its envelope of holomorphy, i.e.:

$$S(D) = S(E(D)) .$$

In view of the difficulty of finding $E(D)$ it may be interesting to calculate $S(D)$ directly.

Furthermore it is not clear whether calculating $S(E(D))$ is always easier than calculating $S(D)$. For the vertex function in configuration space this turns out to be the case. This, however, may be accidental.

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