Commun. math. Phys. 13, 317—334 (1969)

On a Class of Equilibrium States under the Kubo-Martin-Schwinger Boundary Condition

I. Fermions

F. Rocca*,**, M. Sirugue*, and D. Testard
University of Aix-Marseille

Received April 2, 1969; revised form June 10, 1969

Abstract. Using the Kubo-Martin-Schwinger boundary condition for equilibrium states of quantum statistical mechanics of fermion gas, we prove that for $T \neq 0$ a one-particle evolution (corresponding essentially to bilinear hamiltonians) generally defines a unique equilibrium state, which is quasi-free. Conversely any quasi-free state is the equilibrium state for a single one-particle evolution if it has no Fock part in its product decomposition. Limiting cases where $T \to 0$ and $T \to \infty$ are studied. In the case where $T \to 0$ one shows that the state generally converges to a Fock state linked to the evolution.

Introduction

Quasi-free states have been recently studied in an extensive way [1, 2] as possible states for statistical systems of fermions. It is our goal to study the possible dynamics associated with such states.

Our main tool in that study will be the so-called Kubo-Martin-Schwinger boundary condition, in a form given for instance in [3]. This condition has been used in order to derive general properties of equilibrium states under general evolutions. In this paper, we shall restrict ourselves to more specific evolutions, namely the quasi-free evolutions, which are defined in the first section. Nevertheless, some important results of this paper are extended to more sophisticated evolutions.

The second section is devoted to "complexification" of such evolutions; in this way, we are able to define more general automorphisms of a dense subalgebra of the Clifford algebra, which are not *-automorphisms.

We can then prove the main theorem which states that, for a given non-zero temperature, the Kubo-Martin-Schwinger boundary condition establishes a unique correspondence between quasi-free evolutions and quasi-free states which are not of the Fock type. This restriction is quite clear since these states certainly do not satisfy the second condition of Kubo-Martin-Schwinger [3].

---

* Attaché de Recherche au C.N.R.S.

** This work is a part of a "Thèse de Doctorat d'Etat" presented to the "Faculté des Sciences de Marseille", April 23, 1969, under the number A.O. 3073.