Commun. math. Phys. 12, 331—350 (1969)

Harmonic Analysis on the Poincaré Group*

I. Generalized Matrix Elements

NGHIEM XUAN HAI
Laboratoire de Physique Théorique et Hautes Energies**
Orsay, France

Received February 5, 1969

Abstract. The generalized matrix elements of the unitary representations of the Poincaré group are computed as eigendistributions of a complete commuting set of infinitesimal operators on the group. The unitary representations of the Poincaré group can be reconstructed from these matrix elements in a Hilbert space of square integrable functions over the space of eigenvalues. Some properties of these distributions (which are measures) are given and some characters are computed from the explicit formulae for the matrix elements.

Introduction

It was suggested by LURÇAT [1] to construct the quantum fields on the Poincaré group instead of Minkowskian space (which is a homogeneous space of Poincaré group). Numerous attempts were made to enlarge the homogeneous space since then [2, 3]. In the attempt to construct this field theory, it was necessary to obtain the "exponentials" of the Poincaré group, that is to say to begin to study the Fourier transform on the group.

The harmonic analysis on the universal covering group of the proper orthochronous Poincaré group (in this paper this covering group is named improperly Poincaré group) is led trough with the simple idea of using the "exponentials" of the group. These "exponentials" must be the eigenfunctions of the infinitesimal operators and the method is therefore straightforward. In this paper, we first compute the eigenfunctions and in the following one, we shall study the Fourier transform. Chapter 0 is devoted to notations and to some useful definitions.

In Chapter I, we derive the infinitesimal operators (in more mathematical terms: the one-sided invariant vector fields on the group manifold) by using the Lie algebra of the group. We get ten left generators (which are ten independent right invariant vector fields) and ten right generators. In the enveloping algebra we choose a complete subset of commuting operators, which include the two Casimir operators (namely P^2 and W^2), four left operators, and four right operators.

* In partial fulfilment of the requirements for the degree of Docteur d'Etat ès-Sciences Physiques, Faculté d'Orsay, Université de Paris 1969.

** Laboratoire associé au C.N.R.S.