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The Form of Representations of the Canonical Commutation Relations for Bose Fields and Connection with Finitely Many Degrees of Freedom*

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Abstract. Given a representation of the canonical commutation relations (CCR) for Bose fields in a separable (or, under an additional assumption, nonseparable) Hilbert space $\mathfrak{H}$ it is shown that there exists a decreasing sequence of finite and quasi-invariant measures μ_n on the space $\mathscr{V}'$ of all linear functionals on the test function space $\mathscr{V}$, $\mu_1 \geqq \mu_2 \geqq \ldots$, such that $\mathfrak{H}$ can be realized as the direct sum of the $L^2_{\mu_n}$, the space of all μ_n-square-integrable functions on $\mathscr{V}'$. In this realization $U(f)$ becomes multiplication by $e^{i(F,f)}, F \in \mathscr{V}'$. The action of $V(g)$ is similar as in the case of cyclic $U(f)$ which has been treated by ARAKI and GELFAND. But different $L^2_{\mu_n}$ can be mixed now. Simply transcribing the results in terms of direct integrals one obtains a form of the representations which turns out to be essentially the direct integral form of LEW. All results are independent of the dimensionality of $\mathscr{V}$ and hold in particular for $\dim \mathscr{V} < \infty$. Thus one has obtained a form of the CCR which is the same for a finite and an infinite number of degrees of freedom. From this form it is in no way obvious why there is such a great distinction between the finite and infinite case. In order to explore this question we derive von Neumanns theorem about the uniqueness of the Schrödinger operators in a constructive way from this dimensionally independent form and show explicitly at which point the same procedure fails for the infinite case.

A. Introduction

Several branches of quantum field theory are based on the canonical (equal time) commutation relations between the field $\Phi(x)$ and the conjugate field $\pi(x)$. Putting $\boldsymbol{x} = (0, x_1, x_2, x_3)$ one demands

$$[\Phi(\boldsymbol{x}), \pi(\boldsymbol{x}')] = i\,\delta^{(3)}(\boldsymbol{x} - \boldsymbol{x}') \tag{1.1}$$

with the other commutators vanishing.

In order to treat the CCR in a mathematically rigorous manner, one has to regard the fields as operator-valued distributions on a test function space. This can be achieved by replacing $\Phi(\boldsymbol{x})$ and $\pi(\boldsymbol{x})$ by $\Phi(f)$, $\pi(g)$ where f, g are elements of a subspace $\mathscr{V}$ of all real square-integrable

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