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Relatively Compact Interactions in Many Particle Systems

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Abstract. For many-particle relatively compact interactions the essential spectrum of the N-particle Schrödinger Hamiltonian is shown to consist of a continuum $[\Lambda_N, +\infty[$ where Λ_N is the lowest many-body threshold of the system. This result applies in particular to separable interactions and some spin-orbit couplings. The N-particle Green's function is studied with the help of the Weinberg equation for many-body forces whose kernel is shown to be compact in the complementary set of $[\Lambda_N, +\infty[$.

Introduction

It has been shown by HUNZIKER [1,] [2] with the help of the Weinberg equation that the spectrum of a Schrödinger Hamiltonian for a system of particles interacting via two-body forces, consists of a continuum lying in $[\Lambda_N, +\infty[$ where Λ_N is the lowest many-body threshold of the system and in its complement of eigenvalues with finite multiplicities having Λ_N as only possible accumulation point. HUNZIKER's assumption is that the two-body potentials are locally square-integrable and vanish at infinity. Our aim is to extend these results to general "relatively compact" many-particle interactions. Relative compactness is a very useful concept in the study of Schrödinger Hamiltonians, owing on the one hand to the large number of physically interesting local and velocity dependent interactions with this property, and on the other hand to the fact that from the point of view of perturbation theory it enjoys the same virtues as compactness itself. In particular a modified form of Weyl theorem states the invariance of the essential spectrum under such perturbations [5].

As in [2] we shall use the Weinberg equation; the compactness of its kernel will be established with the help of Theorem 1.1 concerning certain connected products of many-particle operators. We shall briefly discuss the relevance of these equations to solve the bound-state problem.

1. Kinematics and Mathematical Preliminaries

We consider a N particle system. We shall denote by $D = \{C_1; C_2; C_k\}$ a decomposition of the system into k disjoint clusters (or fragments) $C_1, \ldots, C_k$. We shall write $\{N\}$ for the trivial decomposition into one

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