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3-Dimensional "Relativity" for Axisymmetric Stationary Space-Times

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Abstract. The equivalence of the axisymmetric stationary vacuum gravitational field problem to a 3-dimensional "relativity theory" in the presence of a certain scalar matter field is shown. An invariant classification can be achieved with respect to the algebraic structure of the 3-dimensional trace-free Ricci-tensor. The extension of these results to electrovac spaces is also discussed.

1. Introduction

H. LEVY [1] found recently a 3-dimensional stress tensor for axially symmetric stationary gravitational fields. The aim of the present work is a further development of this result to a 3-dimensional covariant formulation of the problem.

The basic concept will be discussed in Section 2. We shall show here that the axially symmetric stationary gravitational field problem is in vacuo completely equivalent to a 3-dimensional relativity theory in the presence of a certain "matter field" and with axial symmetry.

A quite general property of the Einstein spaces subject to the condition

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}$$

and containing a Killing vector field is that they can equally well be described as $n-1$ dimensional Einstein spaces, at the expense of appearing some additional material fields. The particular class of symmetric Einstein spaces we picked out excels by the simple structure of the corresponding field equations.

In Section 3 we propose an invariant classification of the related space-times based on the algebraic properties of the 3-dimensional Ricci tensor. Section 4 deals with the electrovac problem. In the Summary we discuss the various new possibilities offered by our method for the study of the axially symmetric space-times.