Commun. math. Phys. 12, 80—90 (1969)

Symmetries Induced by Conserved Vector Currents in the Theory of a Scalar Field

JAN T. ŁOPUSZAŃSKI

Institute of Theoretical Physics, University of Wrocław, Wrocław
Institute of Low Temperature Physics and Structural Research
Polish Academy of Sciences, Wrocław

Received September 19, 1968

Abstract. Let us consider a quantum theory of one scalar, real, local, Poincaré covariant field $A(x)$ with the restricted spectrum condition (massive one particle states and a unique vacuum). The asymptotic fields $A_{\substack{\text{in}\\\text{out}}}(x)$ are assumed to be irreducible. Our conjecture is that under some technical assumptions the "charge" of every real, hermitean, locally conserved, Poincaré covariant quantum (pseudo) vector field $j_\mu(x)$ relatively local to $A(x)$, appearing in this theory-vanishes. This means that in a theory of one scalar, real field with a massive particle one can not expect to get symmetry groups induced by conserved (pseudo) vector currents, only by global, selfadjoint, Poincaré invariant generators.

Our arguments can be easily extended to a theory of one complex scalar field, in this case the only symmetry transformation induced by a current can be the gauge transformation.

We prove also that under very weak assumptions two fields related to each other by a unitary (or similarity) transformation are equal barring some patological cases.

1. Introduction

In many papers concerned with the symmetry problems in quantum field theory (e.g. Goldstone's Theorem and miscellaneous topics related to it) an algebra of quasilocal observables as well as a conserved vector current $j_\mu(x)$ local with respect to the elements of the algebra and to itself are under investigation (see e.g. [1—3]).

It is well known that the necessary condition to get a properly defined "charge" (which in turn gives rise to a one parameter symmetry group of the theory under consideration) is the local conservation of the current $j_\mu(x)$. If the energy-momentum spectrum has a mass gap then it is also a sufficient condition (no — so called — "Goldstone's particles"). The theorems proved up to now state the existence of the "charge". They do not, however, exclude the case that the "charge" vanishes. Of course, such a vanishing "charge" does not give rise to a symmetry.

It is of some interest to learn are we able at all to construct a conserved local current $j_\mu(x)$ out of a scalar, local, irreducible quantum field $A(x)$ in such a way that $j_\mu(x)$ is local with respect to $A(x)$, transforms