Commun. math. Phys. 12, 58—63 (1969)

On the Spectrum of Internal Symmetries in the Algebraic Quantum Field Theory

A. Z. JADCZYK
Institute of Theoretical Physics
University of Wrocław

Received October 14, 1968

Abstract. It is shown that the point spectrum of internal symmetries is always symmetric. It is a group provided the intersection of all local subalgebras is trivial.

1. Introduction

The algebraic quantum field theory is based on the notion of algebras of observables. These are either abstract C^*-algebras [1] or von Neumann algebras of operators [2, 3]. We shall be concerned with these latter (which is quite natural since we intend to deal with conserved symmetries). In the algebraic approach any possible physical symmetry of the quantum system is mathematically represented as an automorphism of the algebra of all observables. The symmetry is conserved, if the vacuum is invariant under the action of unitary operators implementing the associated automorphism. In [4—8] the conditions under which a given symmetry is conserved has been extensively studied in a connection with the Goldstone theorem. We shall be concerned with symmetries which *are* conserved. Space-time translations are such symmetries (by a definition!). These are geometric ones. Aside from them, there also occur symmetries which map each local subalgebra onto itself. They are called internal symmetries. The gauge-type transformations are of this kind.

It should be noticed that, from the purely mathematical point of view, the situation under our consideration is similar to that which occurs in statistical mechanics, when we deal with states of the thermal equilibrium of infinite systems. Consequently, a small modification of the methods used in [11] in studying the spectrum of space and time translations enables us to obtain similar results, in this case, for the spectrum of internal symmetries.

2. General Assumptions and Notation

For the sake of completeness we shall list once more the basic postulates of the algebraic quantum field theory.