Commun. math. Phys. 12, 36—42 (1969)

Properties of Crystal States

H. J. F. Knops

Instituut voor Theoretische Fysika, Universiteit Nijmegen

Received October 24, 1968

Abstract. It is proved that euclidean invariant states describing crystals are not weakly clustering or equivalently that these states exhibit long range order. Further it is shown that a decomposition of an euclidean invariant state into states all of which are invariant for one specific space group, does not yield states with lattice symmetry.

1. Introduction

The purpose of this paper is to investigate states describing crystals. Our method will be that of the algebraic approach to statistical mechanics. In this approach one considers states as positive normalised linear functionals on the quasi-local bounded observables which form a C^*-algebra $\mathfrak{A}$. The euclidean group E^3 is represented as $*$-automorphisms of $\mathfrak{A}$, its action on $A \in \mathfrak{A}$ is denoted by $\alpha_g[A]$, $g \in E^3$.

The representation is moreover supposed to be continuous in the sense:

$$\lim_{g \to e} \|\alpha_g[A] - A\| = 0.$$

As is well known this last property is equivalent with the somewhat more physical requirement of weak continuity i.e. continuity of the functions $\phi(\alpha_g[A])$ for all $A \in \mathfrak{A}$ and all states ϕ [1].

In equilibrium statistical mechanics one starts with a state invariant for the euclidean group and one expects that a state describing a crystal is caracterized by the fact that it can be decomposed into states with lower (crystal) symmetry.

One has proposed several methods to perform this decomposition. Kastler and Robinson [2], restricting themselves to translations, made a decomposition into T_L extremal invariant states, where T_L is the subgroup of translations defined by:

$$T_L = \{\boldsymbol{x} \in R^3 \,|\, \boldsymbol{x} \cdot \boldsymbol{p}_n = 0 \mod 2\pi,\ \boldsymbol{p}_n \in S_D\}$$

S_D being the discrete part of the spectrum of the unitary representation of R^3 corresponding with the state ϕ [2].

Their analysis has been extended by Robinson and Ruelle to the case of the euclidean group [3] (compare also [12]).