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Construction de solutions radiatives approchées des equations d'Einstein

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Abstract. In this paper we construct rigorously, without any averaging scheme, an hyperbolic metric

$$g_{\alpha\beta}(x, \omega\varphi) = \overset{0}{g}_{\alpha\beta}(x) + \frac{1}{\omega}\overset{1}{g}_{\alpha\beta}(x, \omega\varphi) + \frac{1}{\omega^2}\overset{2}{g}_{\alpha\beta}(x, \omega\varphi) \tag{1}$$

where x is a point of the space-time V_4, φ a scalar function on V_4 (the "phase") and ω a great parameter (the "frequency"). This metric is an approximate solution of Einstein's equations: it verifies

$$R_{\alpha\beta} = 0(\omega^{-1}), \tag{2}$$

$\overset{0}{g}_{\alpha\beta}$ and its derivatives, $\overset{1}{g}_{\alpha\beta}$ and $\overset{2}{g}_{\alpha\beta}$ being bounded, the first and second derivatives of $\overset{1}{g}_{\alpha\beta}$, $\overset{2}{g}_{\alpha\beta}$ of order respectively $0(\omega)$ and $0(\omega^2)$.

We first show that the Ricci tensor of (1) stays bounded (when ω increases), with a perturbation $\overset{1}{g}_{\alpha\beta}$ physically significant, if and only if φ verifies the characteristic equation of the back-ground metric and $\overset{1}{g}_{\alpha\beta}$ four algebraic, linear relations, (5.7) and (5.8) in radiative coordinates $x^0 = \varphi$.

We show afterwards that (1) satisfies (2) if $\overset{1}{g}_{\alpha\beta}$ satisfies ordinary differential first order equations (which take a very simple form in radiative coordinates) along the rays of the background, the preceding algebraic relations can be considered as "initial conditions" if the Ricci tensor of the background is of the radiative form

$$R_{\alpha\beta}(\overset{0}{g}_{\lambda\mu}) = \tau\partial_\alpha\varphi\partial_\beta\varphi. \tag{3}$$

It is possible to find $\overset{1}{g}_{\alpha\beta}$ and $\overset{2}{g}_{\alpha\beta}$ with the required conditions of boundedness only if

$$\tau > 0.$$

We apply the results to the Vaydya metric (13.1) and show that, by an oscillatory perturbation of this metric one can satisfy Einstein's equations (to the order ω^{-1}) if the coefficient m usually interpreted as the mass is a decreasing function of $\varphi = u$, which gives, in this context, a proof of the loss of mass by gravitational radiation.

Introduction

Nous nous proposons dans cet article d'utiliser pour l'étude de solutions radiatives des équations d'Einstein[1] une méthode générale de construction d'ondes asymptotiques et d'ondes approchées, dérivée de la

[1] Nous construisons ici des solutions (approchées) des équations du vide, c'est-à-dire des géons gravitationnels au sens de J. A. WHEELER [3]. Une méthode analogue pourrait être utilisée pour des équations des milieux matériels.