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The Damped Self-Interaction

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Abstract. A self-interaction with damped off-diagonal coefficients is used to illustrate techniques for dealing directly with Hamiltonians in strange representations of the CCR.

Introduction

It is clear by now that the existence of many inequivalent representations of the canonical commutation relations (CCR) is both the hope and the bane of the Hamiltonian approach to Quantum Field Theory. Since even the simplest Hamiltonians $\left(\text{for example: } \sum_{k=1}^{\infty} \omega_k(p_k^2 + q_k^2) + a_k q_k + b_k p_k - \lambda_k \text{ if the sequences of real numbers } \{a_k\} \text{ and } \{b_k\} \text{ are large enough}\right)$ do not make sense in the Fock representation one can hope that everything would be all right in another representation. However, the problem of finding the "right" representation and carrying through the analysis of the Hamiltonian in it does not seem, to say the least, to be easy. The usual approach to these problems is first to cut-off the Hamiltonian and develop a well-defined theory on Fock space and then to try to remove the cut-off (using the vacuum expectation values and/or the algebraic approach of Segal) and thereby recover a limiting theory and the "right" representation.

In this note we will sketch how the theory of infinite sums of self-adjoint operators on infinite tensor product spaces developed in [5] and analytic perturbation theory can be used to analyze directly the operator

$$A_\infty = \sum_{k=1}^{\infty} (\omega_k(p_k^2 + q_k^2) - \omega_k \tau_k) + \sum_{k,l,m,n=1}^{\infty} d_{klmn}\, q_k\, q_l\, q_m\, q_n\,.$$

There will be no restriction on the on-diagonal d_{kkkk} except that they be positive, the off-diagonal d_{klmn} must be small. We will find a representation of the CCR such that A_∞ is well-defined and self-adjoint (for an appropriate choice of the renormalizing sequence $\{\tau_k\}$). We show that A_∞ is bounded below and has point spectrum of unit multiplicity as lowest point in its spectrum. We determine sufficient conditions on test functions so that when the field and its conjugate momentum are smeared with them in this representation they are self-adjoint. Finally,