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The Classical Mechanics of One-Dimensional Systems of Infinitely Many Particles

II. Kinetic Theory

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Abstract. We apply the existence theorem for solutions of the equations of motion for infinite systems to study the time evolution of measures on the set of locally finite configurations of particles. The set of allowed initial configurations and the time evolution mappings are shown to be measurable. It is shown that infinite volume limit states of thermodynamic ensembles at low activity or for positive potentials are concentrated on the set of allowed initial configurations and are invariant under the time evolution. The total entropy per unit volume is shown to be constant in time for a large class of states, if the potential satisfies a stability condition.

§ 1. Introduction

In [1], we proved an existence and uniqueness theorem for solutions of the equations of motion for systems of infinitely many particles. In this article, we will apply this theorem to the study of the time-evolution of states of classical statistical mechanics. Let us recall briefly the notation and results of [1]. We denote by $\mathscr{X}$ the set of locally finite configurations of labelled particles and by $[\mathscr{X}]$ the corresponding set of configurations of unlabelled particles. A state of classical statistical mechanics is a probability measure on $[\mathscr{X}]$ invariant under space translations. Let $\hat{\mathscr{X}}$ denote the set of labelled configurations satisfying conditions 1) and 2) of [1]. Theorem 2.1 of [1] asserts the existence of a solution of the equations

$$\frac{dq_i(t)}{dt} = p_i(t)$$

$$\frac{dp_i(t)}{dt} = \sum_{j \neq i} F(q_i(t) - q_j(t))$$

and the initial conditions

$$q_i(0) = q_i\,, \qquad p_i(0) = p_i\,,$$

provided that F has compact support and satisfies a Lipschitz condition and that the initial configuration (q_i, p_i) is in $\hat{\mathscr{X}}$; it also asserts the uni-

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