

Boson Fields with the  $:\Phi^4:$  Interaction  
in Three Dimensions\*

JAMES GLIMM

Massachusetts Institute of Technology  
Cambridge, Mass.

Received April 4, 1968

**Abstract.** The  $:\Phi^4:$  interaction for boson fields is considered in three dimensional space time. A space cutoff is included in the interaction term. The main result is that the renormalized Hamiltonian  $H_{\text{ren}}$  is a densely defined symmetric operator. In addition to the infinite vacuum energy and infinite mass renormalizations, this theory has an infinite wave function renormalization. Consequently the Hilbert space (of physical particles) in which  $H_{\text{ren}}$  acts is disjoint from the bare particle Fock Hilbert space in which the unrenormalized Hamiltonian is defined.

Contents

1. Introduction . . . . .	2
1.1 Superrenormalizable problems . . . . .	2
1.2 The Domain for $H_{\text{ren}}$ . . . . .	2
1.3 Infinite Renormalizations . . . . .	4
1.4 The Unrenormalized Hamiltonian . . . . .	5
1.5 Products and Their Graphs . . . . .	6
2. Products of $IV_j$ 's . . . . .	10
2.1 Introduction . . . . .	10
2.2 Estimates on Products . . . . .	14
3. The Dressing Transformation $T$ . . . . .	25
3.1 Introduction . . . . .	25
3.2 The Definition of $T$ . . . . .	25
3.3 The Renormalized Inner Product . . . . .	27
4. The Definition of $H_{\text{ren}}$ . . . . .	37
4.1 Introduction . . . . .	37
4.2 Finite Contributions to $H_{\text{ren}}$ . . . . .	39
4.3 Renormalizing the Creation Part of $V$ . . . . .	40
4.4 Renormalizing the Annihilation Part of $V$ . . . . .	43
4.5 Renormalizing the Self Energy . . . . .	45

\* This work was supported in part by the National Science Foundation, NSF GP 7477.