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Quantum Theory of Gravitation and Locality Postulate*

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Abstract. A quantized space-time metric $g_{ik}(x)$ is investigated within a suitably modified axiomatic approach. Coordinate distances dx are called absolutely space-like if $g_{ik}(x)\, dx^i\, dx^k$ is negative definite. For such distances, fields are assumed to commute or anticommute, respectively (generalized locality). The quantum fluctuations of the light cone $g_{ik}(x)\, dx^i\, dx^k = 0$ are shown to extend to distances dx which are space-like with respect to the Minkowski metric. Generalized locality is therefore weaker than the usual locality postulate.

I. Quantized Gravity and Wightman Axioms

A heuristic attempt is presented to describe a quantized space-time metric (i.e., quantized gravitational potentials) $g_{ik}(x)$ according to general ideas of axiomatic field theory. For this purpose, a considerably weakened form of the usual field theory axiomatics (compare, e.g., STREATER and WIGHTMAN [1]) is appropriate. A closely related approach to the quantum theory of gravitation has been proposed independently by STROCCHI [2]. The assumptions used here will be formulated and discussed with respect to their applicability to quantized gravity. Most problematic in this respect is the usual locality postulate, which is therefore treated separately in the next section.

Postulate 1 (Quantum Theory, Invariance, Spectrum). Pure states of the system are unit vectors Ψ in a Hilbert space $\mathscr{H}$ with positive norm. The inhomogeneous Lorentz group (or, more precisely, its covering group) is continuously represented in $\mathscr{H}$ by unitary operators $U(a, \Lambda)$. With $U(a, 1) = e^{iP_k a^k}$, the spectrum of P_k is contained in the closed forward light cone. A vacuum state Ψ_0, defined by $U(a, \Lambda)\,\Psi_0 = \Psi_0$, exists and is unique. –

This postulate becomes meaningful only with a suitable interpretation of $U(a, \Lambda)$, since a representation $U(a, \Lambda)$ with the required properties can be constructed formally in any infinite-dimensional Hilbert space $\mathscr{H}$.

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