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A Singularity Free Solution of the Maxwell-Einstein Equations

P. Dolan
Department of Mathematics, Imperial College, London

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Abstract. It is found that the massless charged particle permanently at rest at the origin of spherical polar coordinates in Lovelock's interpretation [1] of Robinson's solution of the Einstein-Maxwell equations [2] will repel all charged test particles, irrespective of the sign of their charges. By a global embedding of the space-time in a flat 6-space we find an absence of singularities where point-charges or point-masses might be located. With the use of the Newman-Penrose method of spin-coefficients [6] it is shown that all the Robinson solutions [2] represent constant electromagnetic fields.

1. Introduction

The Einstein-Maxwell equations in the absence of sources are

$$\left.\begin{aligned} & R^i_k - \frac{1}{2} R\delta^i_k + F^{ij} F_{jk} + {}^*F^{ij}\, {}^*F_{jk} = 0 \,, \\ & F^{ij}_{\;;j} = 0 \,, \quad {}^*F^{ij}_{\;;j} = 0 \,, \end{aligned}\right\} \tag{1.1}$$

where R^i_k is the Ricci tensor, $F_{ij} = -F_{ji}$ the electromagnetic field tensor and ${}^*F_{ij} = 1/2\,(-g)^{1/2}\,\varepsilon_{ijkl} F^{kl}$ its dual. Covariant differentation with respect to the metric tensor g_{ij} is denoted by a semi-colon. Robinson [2] has found the following solution of (1.1):

$$\left.\begin{aligned} ds^2 &= \left(\frac{e^2}{r^2}\right)(c^2\,dt^2 - dr^2 - r^2\,d\theta^2 - r^2 \sin^2\theta\, d\phi^2)\,, \\ F_{ij} &= e(\xi_{ij}\cos\beta + {}^*\xi_{ij}\sin\beta)\,, \\ \xi_{ij} &= \left(\frac{1}{r^2}\right)\delta^{10}_{ij} \end{aligned}\right\} \tag{1.2}$$

where e and β are disposable constants.

The solution (1.2), if written in terms of the new coordinates

$$\tilde{x}^0 = e(ct - r)\,,\ \tilde{x}^1 = \frac{e}{r}\,,\quad \tilde{x}^2 = e(\pi/2 - \theta)\,,\quad \tilde{x}^3 = e\phi\,,$$

becomes

$$\left.\begin{aligned} ds^2 &= e^{-2}(\tilde{x}^1\,d\tilde{x}^0)^2 + 2\,d\tilde{x}^0\,d\tilde{x}^1 - (d\tilde{x}^2)^2 - (\cos(\tilde{x}^2/e)\,d\tilde{x}^3)^2 \\ F_{ij} &= \frac{1}{e}\,\delta^{01}_{ij}\cos\beta + \frac{1}{e}\,\delta^{23}_{ij}\cos(\tilde{x}^2/e)\sin\beta \end{aligned}\right\} \tag{1.3}$$

in which the Maxwell field is now expressed independently of the coordinate r [2]. A further coordinate transformation

$$2cT = \tilde{x}^0 + \tilde{x}^1\,,\quad 2X = \tilde{x}^0 - \tilde{x}^1\,,\quad Y = \tilde{x}^2\,,\quad Z = \tilde{x}^3$$